Accounting for Under Reporting in Disease Counts

The presence only problem

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The presence only problem

The presence only problem

- Information is only known about the presences
- The observed counts are only a subset of the population
- Often encountered in epidemiology, ecology, criminology etc
- It is problematic when we want to estimate population prevalence/incidence

How can we account for this in our estimation procedure?

The Likelihood

The observed counts Y are only a subset of the true number infected Z, from population N:

$$Z \sim Binomial(N, \lambda)$$

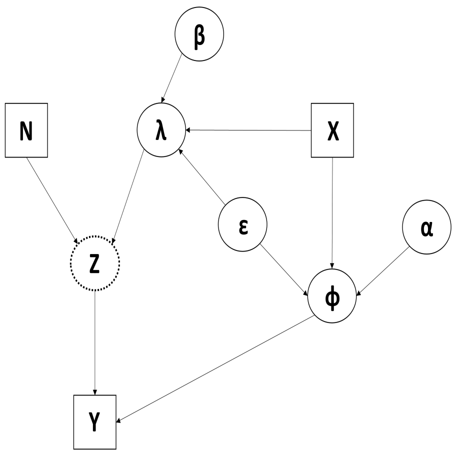
$$Y \sim Binomial(Z, \phi)$$

$$\begin{aligned} f(Y|N, \lambda, Z, \phi) &= \sum_{Z} \binom{N}{Z} \lambda^{Z} (1-\lambda)^{N-Z} \binom{Z}{Y} \phi^{Y} (1-\phi)^{Z-Y} \\ &= \binom{N}{Y} (\lambda\phi)^{Y} (1-\phi\lambda)^{N-Y} \end{aligned}$$

$$Y \sim Binomial(N, \lambda\phi)$$

The only way is Bayes!

Bayesian Hierarchical Models



$$Y \sim Binomial(N, \lambda(X)\phi(X))$$

$$\lambda(X) = \frac{exp\{X\beta + \varepsilon\}}{1 + exp\{X\beta + \varepsilon\}}$$

$$\phi(X) = \frac{exp\{X\alpha + \varepsilon\}}{1 + exp\{X\alpha + \varepsilon\}}$$

- α and β are vectors of regression coefficients
- ε is a spatial residual, with CAR structure

Posterior Distribution

The joint posterior distribution:

$$f(\lambda, \phi | Y, N) \propto f(Y | N, \lambda, \phi) f(\lambda) f(\phi).$$

The posterior distribution for Z:

$$f(Z | \lambda, \phi, Y, N) = \int_0^1 \int_0^1 \binom{N}{Z} \lambda^Z (1-\lambda)^{N-Z} \binom{Z}{Y} \phi^Y (1-\phi)^{Z-Y} f(\lambda) f(\phi) d\phi d\lambda.$$

Simulation Studies

The Scenarios

Underlying Prevalence, $\lambda(X)$:

$$\beta_0 = \{-6.7, -2\}, \beta_1 = \{0, 2.5\}$$

Detection ϕ:

$$\phi = \{0.7, 0.9\}$$

Presence of spatial autocorrelation ε:

$$\varepsilon = \{\varepsilon_1, \varepsilon_2\}$$

with ε_1 indicating spatial autocorrelation and white noise otherwise.

- covariates are considered only for λ
- ϕ is treated as an intercept only model.

In this 2^4 different scenarios were investigated.

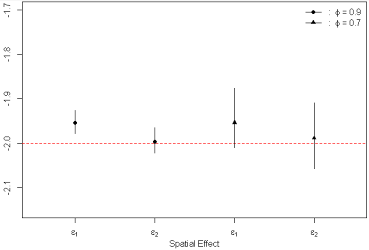
Priors

- $\beta_0, \beta_1 \sim N(0, 0.04)$
- $\phi|\phi = 0.9; \alpha \sim N(logit(0.9), 100)$
- $\phi|\phi = 0.7; \alpha \sim N(logit(0.7), 100)$
- $\varepsilon_{ij} \sim N(\bar{\varepsilon}_{-j}, \tau m_j)$

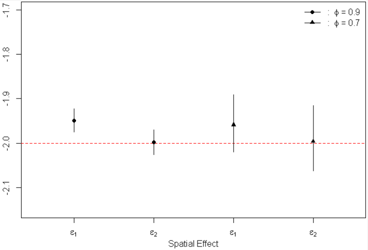
Identifiability

- Placing an informative prior on ϕ was necessary for identifiability and convergence of the model

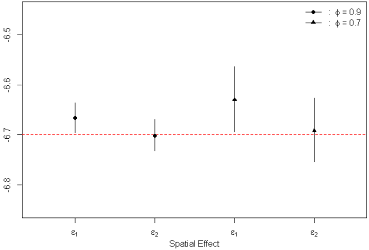
Results



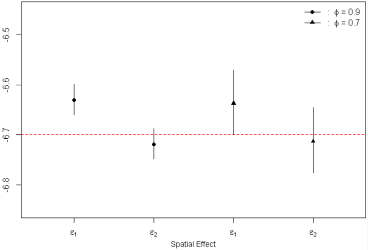
(a) $\hat{\beta}_0|\beta_0 = -2, \beta_1 = 0$



(b) $\hat{\beta}_0|\beta_0 = -2, \beta_1 = 2.5$

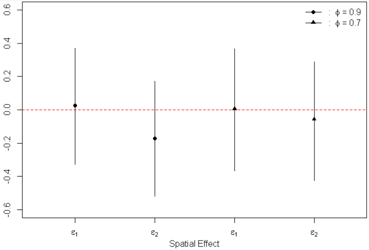


(c) $\hat{\beta}_0|\beta_0 = -6.7, \beta_1 = 0$

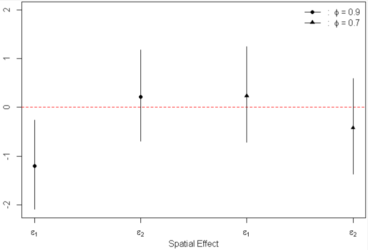


(d) $\hat{\beta}_0|\beta_0 = -6.7, \beta_1 = 2.5$

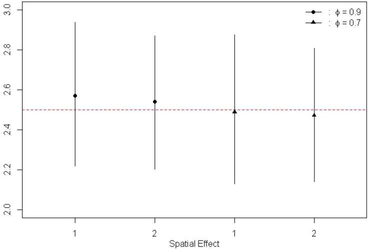
Results



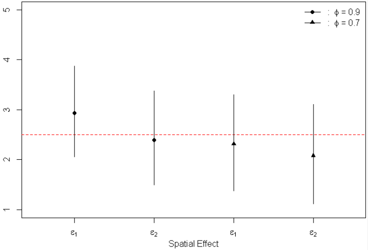
(a) $\hat{\beta}_1|\beta_0 = -2, \beta_1 = 0$



(b) $\hat{\beta}_1|\beta_0 = -6.7, \beta_1 = 0$



(c) $\hat{\beta}_1|\beta_0 = -2, \beta_1 = 2.5$



(d) $\hat{\beta}_1|\beta_0 = -6.7, \beta_1 = 2.5$

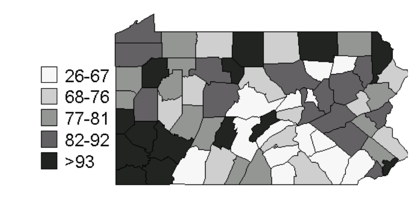
Pennsylvania Lung Cancer Data Set

Pennsylvania Lung Cancer Data Set

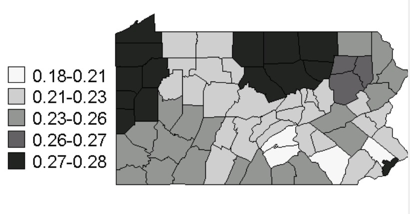
- Pennsylvania Lung Cancer Data available from the SpatialEpi package in R.
- Comprises of lung cancer cases and population counts at the county level, with $n = 67$.
- County-specific smoking rates.
- Population counts were obtained from the 2000 decennial census
- Stratified on race (white vs non-white), gender and age (Under 40, 40-59, 60-69 and 70+).

For simplicity, we aggregated the data to county specific level only.

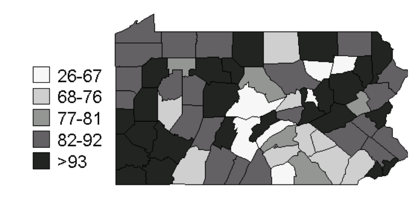
Pennsylvania Lung Cancer Data Set: Results



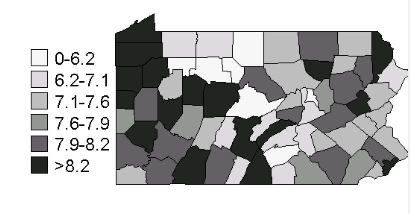
(a) Observed Prevalence



(b) Proportion of Smokers



(c) Predicted Prevalence



(d) Predicted - Observed

Conclusion and Future Work

Summary

- Can be used to estimate true population parameters
- Potential to uncover areas of under detection/ under reporting

Future Work

- Application to kidney stones data set
- Investigate the correlations between hierarchies
- Covariates on detection

Questions?